Q7 11.4

Determine the convergence/divergence of the following series. Show all work neatly with **clear presentations** including naming any tests used and showing all requirements have been met. Note: tests from previous sections may be helpful on some.

1) $\sum_{n=1}^{\infty} \frac{2+\sin n}{4^n}$ Compare to $\sum \frac{3}{4^n}$ which converge (geometric, $r=\frac{1}{4}<1$)

since $-1 \le \sin n \le 1$

$0 \le 2+\sin n \le 3$

$0 \le \frac{2+\sin n}{4^n} \le \frac{3}{4^n}$

So by comparison test

$\sum_{n=1}^{\infty} \frac{2+\sin n}{4^n}$ converges

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2) $\sum_{n=1}^{\infty} \frac{\ln n}{n^3+1}$ Compare to $\sum \frac{1}{n^2}$ which is a convergent p series ($p=2>1$)

Since $0 \le \ln n < n$ for $n \ge 1$

$0 \le \frac{\ln n}{n^3+1} < \frac{n}{n^3+1} < \frac{n}{n^3} = \frac{1}{n^2}$

So by comparison test,

$\sum_{n=1}^{\infty} \frac{\ln n}{n^3+1}$ converges

(show condition of the test used is satisfied)

3) $\displaystyle\sum_{n=1}^{\infty} \frac{3n}{\sqrt{4n^2+3}}$

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{3n}{\sqrt{4n^2+3}} \cdot \frac{\sqrt{1/n^2}}{\sqrt{1/n^2}} = \lim_{n\to\infty} \frac{3}{\sqrt{4+3/n^2}} = \frac{3}{2}$$

Since $\lim_{n\to\infty} a_n \neq 0$, the series

$\displaystyle\sum_{n=1}^{\infty} \frac{3n}{\sqrt{4n^2+3}}$ diverges by Test for divergence

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4) $\displaystyle\sum_{n=1}^{\infty} \frac{n-3}{n\sqrt{n}}$

Compare to $\sum \frac{1}{\sqrt{n}}$ which is a divergent p series, $p = 1/2 < 1$

Unfortunately,

$$\frac{n-3}{n\sqrt{n}} \cancel{>} \frac{n}{n\sqrt{n}} = \frac{1}{\sqrt{n}}$$

Thats what would need to use the comparison test

so apply Limit comarison test $\left(\frac{n-3}{n\sqrt{n}} > 0 \text{ for } n > 3\right)$

$$c = \lim_{n\to\infty} \frac{\frac{n-3}{n\sqrt{n}}}{\frac{1}{\sqrt{n}}} = \lim_{n\to\infty} \frac{(n-3)\sqrt{n}}{n\sqrt{n}} = \lim_{n\to\infty} \frac{n-3}{n} = 1$$

Since c is finite and nonzero, and since $\sum \frac{1}{\sqrt{n}}$ diverges, so must $\sum \frac{n-3}{n\sqrt{n}}$ by LCT